

North Fulton Varsity Math Tournament 2008

Hosted by Alpharetta High School & Northview High School

Multiple Choice Exam

Please do not open this packet until instructed to do so by your proctor. This is a 60-minute exam, consisting of 40 multiple-choice questions. You will receive 4 points for every correct answer and lose 1 point for every incorrect answer. **NO CALCULATORS** will be allowed on any portion of this exam. Please make sure your name and school are written on your scantron. We will not grade any exam that is missing this information. Good luck!

- The average of 4 numbers is 12. If you remove one of the numbers, the average of the remaining 3 is 7. What number was removed?
 - 48
 - 24
 - 21
 - 27
 - 16
- How many rooks can be placed on a 13-by-13 board so that no two rooks threaten each other? (Rooks threaten each other vertically and horizontally across a chessboard)
 - 26
 - 8
 - 13
 - 169
 - 12
- Given an isosceles trapezoid with bases of length 18 and 36 and area $243\sqrt{3}$, find the length of the trapezoid's diagonal.
 - 9
 - $9\sqrt{3}$
 - 27
 - 18
 - $18\sqrt{3}$
- Find the infinite sum: $1 + \frac{2}{7} + \frac{3}{49} + \frac{4}{343} + \dots$
 - $\frac{49}{36}$
 - $\frac{4}{3}$
 - $\frac{7}{6}$
 - $\frac{6}{2}$
 - 1
- Given that an alphabet is composed of 7 distinct letters, how many four letter "words" can be formed? Assume that a "word" is any combination of letters, excluding repetitions, regardless of a dictionary definition.
 - 840
 - 80
 - 35
 - 210
 - 28
- Let $f(x) = 3x^5 + bx^4 + 8x^3 + 10x^2 + ex + f$. Given $f(0) = 3$, $f(-1) = -7$, and $f(1) = 23$, what is b ?
 - 5
 - 10
 - 5
 - 12
 - 16

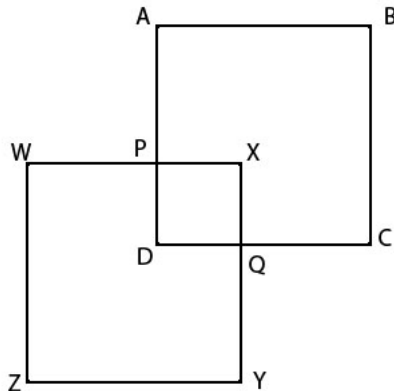
7. What fraction of the divisors of 2008 are both even and not prime?

- (A) $\frac{1}{4}$
- (B) $\frac{1}{8}$
- (C) $\frac{1}{16}$
- (D) $\frac{1}{32}$
- (E) $\frac{1}{64}$

8. A survey was taken of 100 Alpharetta High School students on sea creatures they would keep as pets. 33 students said they would keep a jellyfish, 40 said a sea cucumber, and 42 said an octopus would make a good pet. If 7 wanted all three as pets, 12 wanted both octopi and jelly fish, 20 wanted a jellyfish and sea cucumber, and 18 wanted both octopi and sea cucumbers, how many students did not want any of the animals as pets?

- (A) 19
- (B) 13
- (C) 28
- (D) 9
- (E) 49

9. Two squares of side length 4 intersect as shown, forming square PXQD. If the distance from Z to D is $3\sqrt{2}$, what is the area of PBQZ?



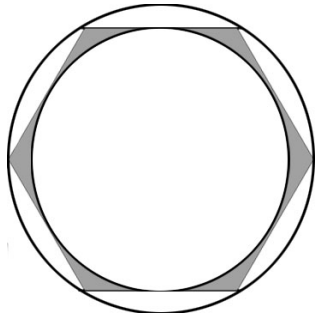
- (A) 1
- (B) 6
- (C) $4\sqrt{2}$
- (D) 2
- (E) 7

10. The probability that a student gets into a car accident in the Alpharetta High School parking lot is $\frac{3}{10}$. Out of 7 seniors with parking passes, what is the probability that over four of these students or none at all will get into a car accident? Round to the nearest thousandth.

- (A) .029
- (B) .001

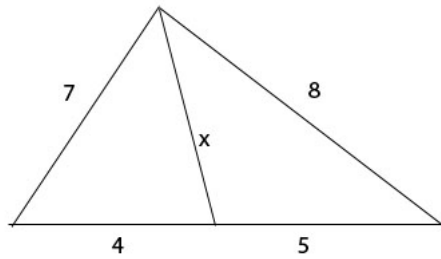
- (C) .082
 (D) .111
 (E) .9
11. Given that $x + y = 7$ and $x^2 + y^2 = 22$, what is that value of $\frac{1}{x} + \frac{1}{y}$?
- (A) $\frac{1}{49}$
 (B) $\frac{7}{27}$
 (C) $\frac{7}{7}$
 (D) $\frac{14}{15}$
 (E) $\frac{14}{27}$
12. On planet Gamma, there are only \$8 bills, \$13 bills, and \$15 bills. If Krishna owns a taffy store on planet Gamma, what is the highest price she cannot sell the taffy for?
- (A) 43
 (B) 83
 (C) 35
 (D) 14
 (E) 42
13. How many 5-digit numbers are also palindromes?
- (A) 1001
 (B) 1000
 (C) 900
 (D) 901
 (E) 950
14. Find the value of: $\sin^2 13^\circ + \sin^2 33^\circ + \sin^2 57^\circ + \sin^2 77^\circ$
- (A) 2
 (B) $\sqrt{2}$
 (C) $1 + \sqrt{3}$
 (D) 1
 (E) $2\sqrt{2}$
15. Aaron is packing perfectly spherical steaks into a box. Clearly, with his love of steak, he wants to optimize the size of the box. Thus, he wants to place the steaks in a box with as small of a volume as possible. If the radius of each steak is 1 inch, what is the volume of the box?
- (A) $32 + 32\sqrt{2}$
 (B) $6 + 4\sqrt{2}$
 (C) $20 + 14\sqrt{2}$
 (D) $2\sqrt{2}$
 (E) $2 + 2\sqrt{2}$
16. If $123_{10} = 323_n$, what is the value of 131_n ?
- (A) 71
 (B) 6
 (C) 7

- (D) 43
(E) 55
17. What is the remainder when $2008!$ is divided by 7?
(A) 2
(B) 0
(C) 1
(D) 6
(E) 4
18. Find the sum of all roots (both real and imaginary) of the equation $7x^4 + 3x^3 - 2x^2 + 4x - 13 = 0$.
(A) $-\frac{3}{7}$
(B) -3
(C) $-\frac{13}{7}$
(D) $-\frac{4}{7}$
(E) 13
19. Let $f(n)$ be a function such that $f(n) = 2 - 3f(2008 - n)$. What is the value of $f(2008)$?
(A) -4
(B) $-\frac{1}{2}$
(C) 8
(D) $\frac{1}{2}$
(E) -1
20. Bessie, the friendly Alpharetta High School cow, is tied to the outside corner of a fence around the practice field (which is 360 ft long by 160 ft wide). Assuming Bessie's rope is 200 ft long, what is the size of the area that Bessie is free to roam?
(A) 30400π
(B) 40000π
(C) 21000π
(D) 30000π
(E) 27500π
21. Which of the following statements are equivalent to $(A' \cap ((A \cup A') \cap B))' \cap (A \cup B)$? Assume I represents the Universe and \emptyset represents the Empty Set.
(A) I
(B) A
(C) $A \cap B$
(D) B
(E) $A \cup B$



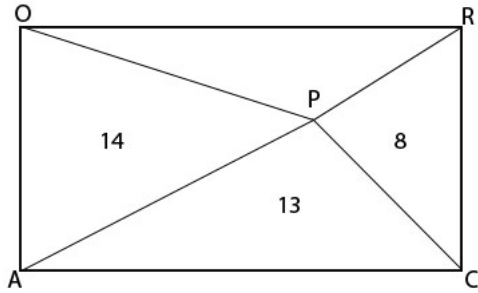
22. A regular hexagon is inscribed inside a circle. Another circle is inscribed within that hexagon. Find the ratio of the area of the shaded region to that of the larger circle.

- (A) $\frac{3\sqrt{3}}{2} - \frac{3}{4}\pi$
 (B) $\frac{3\sqrt{2}}{\pi} - \frac{3}{2}$
 (C) $\frac{3\sqrt{3}}{2}$
 (D) $\frac{3}{4}\pi$
 (E) $\frac{3\sqrt{3}}{2\pi} - \frac{3}{4}$



23. Find x .

- (A) 10
 (B) $\frac{\sqrt{321}}{3}$
 (C) $\frac{\sqrt{163}}{2}$
 (D) $\frac{42}{5}$
 (E) 7

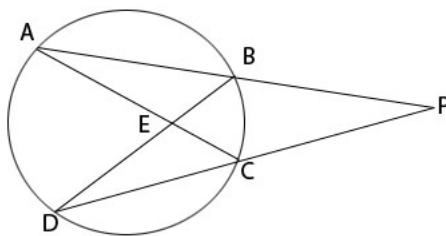


24. Find the area of $\triangle OPR$.

- (A) $\frac{91}{4}$
- (B) 6
- (C) 10
- (D) 9
- (E) $\frac{9}{2}$

25. What is the coefficient of x^3 in the expansion of $(\sqrt{x} + \sqrt{2})^9$?

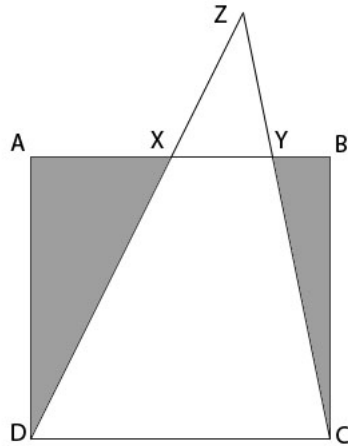
- (A) 147
- (B) 168
- (C) $168\sqrt{2}$
- (D) $2\sqrt{2}$
- (E) 84



26. If $\widehat{AD} = 100^\circ$, and $\angle BPC = 30^\circ$, Find the sum of \widehat{AB} and \widehat{CD} . Note that

- \widehat{AD} goes counterclockwise from A to D.
- (A) 120°
 (B) 100°
 (C) 140°
 (D) 220°
 (E) 165°
27. The solution to the following system of equations can be written as (x, y) .
 Find $x + y$.
 $\log_{16}(\log_4(\log_2(4x + 8))) = 0$
 $\log_{12}(\log_{4x}(7y - 6)) = 0$
- (A) 16
 (B) 14
 (C) 4
 (D) 7
 (E) 8
28. $\log_x y$, $\log_y x$, and 64 are the first three terms of a geometric sequence. If x is equal to 81, find y .
- (A) 16
 (B) 7
 (C) 12
 (D) 9
 (E) 3
29. What is the largest angle formed by extending two sides of a regular dodecagon?
- (A) 100°
 (B) 120°
 (C) 130°
 (D) 135°
 (E) 150°
30. Given that the roots of the equation $x^2 + bx + c = 0$ are -4 and 2 , find the value of the equation's discriminant.
- (A) $2\sqrt{5}$
 (B) 36
 (C) $2\sqrt{2}$
 (D) 8
 (E) -20

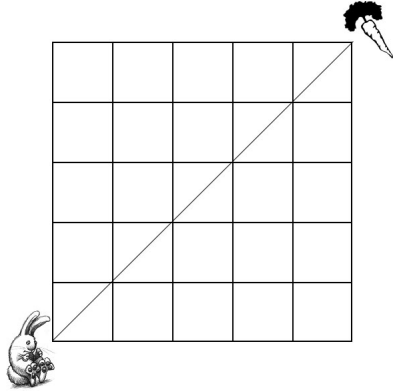
31. How many times in a 24-hour period are the minute hand and hour hand of a clock at a right angle to each other?
- (A) 44
 (B) 24
 (C) 48
 (D) 25
 (E) 46



32. Given $ABCD$ is a square of side length 5, and the height of $\triangle XYZ$ is 1, find the area of the shaded region.
- (A) $\frac{125}{12}$
 (B) 15
 (C) $\frac{72}{5}$
 (D) 10
 (E) $\frac{3}{5}$
33. Find the value of $\frac{2}{1 + \frac{2}{1 + \frac{2}{\dots}}}$.
- (A) -2
 (B) 2
 (C) 1
 (D) 3
 (E) $\sqrt{2}$
34. The sum of 5 consecutive odd integers is 235. What is the product of the smallest and largest of these integers?
- (A) 94
 (B) 2205
 (C) 99
 (D) 2193
 (E) 2295

35. Angela is 25 years younger than her mother. In two years, she will be half the age her mother was when Angela was the age of her brother now. Assuming her brother is 8 years younger than her, what is the sum of all their ages?
- (A) 13
 (B) 43
 (C) 56
 (D) 38
 (E) 18
36. The hexadecimal system is a base-16 number system where digits are composed of the numbers 0-9 as well as the letters A,B,C,D,E, and F to represent the values of 10,11,12,13,14, and 15 respectively. Express the hexadecimal number $5D1$ in base 2.
- (A) 11111010001
 (B) 10101010101
 (C) 10001011101
 (D) 10111000001
 (E) 10111010001
37. Vince, ever the responsible math team captain, is leading a group of timid mathletes through the dangerous streets of Savannah. He is attempting to cross the main street when he realizes a semi truck is blocking his way. He also notices that the semi is followed by one normal car, then another truck, then two normal cars and another truck, then three normal cars, and another truck, etc. If Vince decides to run underneath each semi in a zig-zag pattern, passing cars on the one side then ducking under a truck and passing cars on the other. If, in his courageous battle to conquer the machine, Vince runs under 30 trucks total, how many cars does he pass on this excursion?
- (A) 427
 (B) 415
 (C) 465
 (D) 325
 (E) 435
38. On a 25-question multiple choice test with no penalty for guessing, the probability that a normal person answers a question correctly is $\frac{2}{5}$. Kevin Yang is really lucky. Thus, his probability of getting a question right is actual $\frac{4}{5}$. What is the difference between the probability that Kevin Yang gets at least 10 questions right and the probability that a normal person gets at least 10 questions right?
- (A) $\frac{2^{10}-1}{5^{10}}$
 (B) $\frac{2^{10}(2^{10}-1)}{5^{10}}$
 (C) $(\frac{4}{5})^{10}$
 (D) $\frac{1}{5^{15}}(\frac{4}{5})^{10} - (\frac{3}{5})^{15}(\frac{2}{5})^{10}$

(E) $\frac{1}{5^{15}} - \left(\frac{3}{5}\right)^{15}$



39. Assume that a bunny is at the bottom left corner of the grid to the left. It wants to get to the carrot in the top right corner, but it can only go to the right and up along the edges of the grid. Assuming it cannot cross the diagonal drawn, how many paths can it follow to the carrot?
- (A) 23
 (B) 41
 (C) 14
 (D) 42
 (E) 35
40. How many ways are there to rearrange the letters in MAJA WICHROWSKA into two words of length 4 and 10?
- (A) $\frac{14!}{3!2!}$
 (B) $\frac{14!}{10!4!}$
 (C) $\frac{4!}{2!} + \frac{10!}{2!}$
 (D) $\frac{2 \cdot 14!}{3!2!}$
 (E) $\frac{4!10!}{2!3!}$